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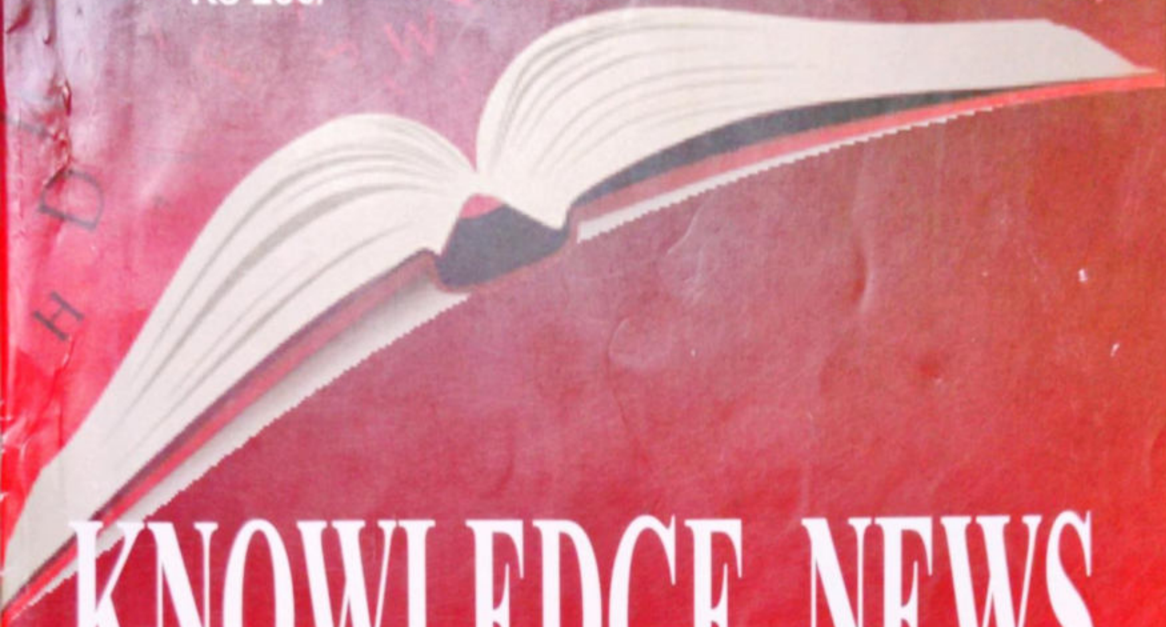

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ANALYZING THE COMPETENCY OF MATHEMATICAL MODELING IN PHYSICAL SCIENCES

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Abstract

A primary goal of modern science is to create mathematical models that allow both predictions and explanations of physical phenomena. We weave mathematics extensively into our modern science instruction beginning in high school, and the level and complexity of the mathematics we draw on grows as our modern researcher's progress through a modern science curriculum. Despite much research on the learning of both modern science and math, the problem of how to successfully teach most of our students to use mathematics in modern science effectively remains unsolved. A fundamental issue is that in modern science, we don't just use mathematics, we think about the physical world with it. As a result, we make meaning with mathematical symbology in a different way than mathematicians do. In this talk we analyze how developing the competency of mathematical modeling is more than just "learning to do math" but requires learning to blend physical meaning into mathematical representations and use that physical meaning in solving problems. Examples are drawn from across the curriculum.

Keywords: Modern science education research, mathematics in science, making meaning with mathematics .

1. Introduction

Mathematics plays a significant role in modern science instruction, even in introductory classes, but not always in a way that is successful for all students. As modern science students learn the culture of modern science and grow from novice to expert, many have trouble bridging what they learn in math with how we use mathematics in modern science. As instructors, many of us are distressed and confused when our students succeed in mathematics classes but fail to use those same tools effectively in modern science. Part of the difficulty is that in modern science, we don't just calculate with mathematics, we "make meaning" with it, think with it, and use it to create new modern science. [7]

Mathematics has been identified as a critical scientific competency both by the European Union (EUR-LEX 2006) and the US biology community (National Research Council 2003, AAMC/HHMI 2009, AAAS 2011), so as we think about how we might improve modern science instruction it is important to try to understand what role mathematics play in modern science, how that role may be difficult for students, and how we might learn to think about that difficulty. A crucial element is the role that mathematics plays in the epistemology of modern science.

The process of science and the development of scientific thinking is all about epistemology – deciding what we know and how we decide that we know it. In modern science, mathematics has been closely tied with our epistemology for 300 years, transforming modern science from natural philosophy into the mathematical science it is today.

Since we have been trying to develop insight into what is going on in our students' minds, our data is mostly qualitative. It often involves videos of problem-solving interviews or ethnographic data of students in real classes solving real homework problems, either alone

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or in groups. Sometimes we have quantitative data as well, including responses of many students on multiple choice questions on exams or with clickers in a large lecture class.

We work in the theoretical framework of Resources – the idea that student thinking is highly dynamic, calling on multiple smaller bits of knowledge that may be organized in locally coherent, but often changing ways. (Hammer 2000) This framework is built on ideas from education, psychology, neuroscience, sociology, and linguistics research. (Redish 2014)

We often say that "mathematics is the language of modern science", but what physicists do with mathematics is deeply different from what mathematicians do with it. Mathematicians and physicists load meaning onto symbols differently and this has profound implications. (Redish & Kuo 2015).

In modern science, we link our equations to physical systems and this adds information on how to interpret them. Our symbols carry extra information not present in the abstract mathematical structure of the equation. As a result, our processing of equations in modern science has additional levels and may be more complex than the processing of similar equations in a math class.

In modern science most of our symbols don't stand for numbers (or collections of numbers) but for measurements. Our symbols bring physical properties along with them. As a result, they have units that depend on the measurement process. In math terms, this is quite sophisticated. As a result of the arbitrariness in our choice of units, modern science equations must have a particular structure. Since the choice of scale is arbitrary, any physically true relation must be true whatever choice of scale is made. This means that every part of both sides of the equation must change in the same way when a scale is changed. Mathematically this means that equations must trans-form properly, covariant by an irreducible representation of the 3-parameter scaling group SxSxS for units of mass, length, and time. (Bridgman 1922).

What about significant figures? Why do we bother talking about them now that we have calculators? But when we multiply 5.42 x 8.73 in a 6th grade arithmetic class we want something different from what we want when we are measuring the area of a (5.42 cm) × (8.73 cm) sheet of silicon. Every physical measurement has an uncertainty that propagates to the product, leaving many digits shown by a calculator as "insignificant figures", irrelevant to physical silicon.

A second example illustrates another of the differences between mathematics and modern science classes. Mathematics classes typically use equations with a small number of symbols, with fixed conventions for what symbols stand for variables and what for constants. Furthermore, introductory mathematics classes (through calculus) often do very little with parameter dependence. In modern science, on the other hand, our equations often involve a blizzard of symbols, some of which may be variables or constants depending on what problem we choose to consider. An example occurred in an introductory modern science class for life scientists. One year of calculus was a pre-requisite and most of the students in the class had earned a good grade in that class.

2. Mathematical Model

Our examples suggest that the critical difference in mathematics-as-pure-mathematics and mathematics-in-a-modern science-context is the blending of physical and mathematical knowledge. A simple model (Re-dish 2005) focuses on a few of the main steps: (1) choosing a model to map physical quantities into mathematical structures, (2) processing, using the tools inherited from those mathematical structures, (3) interpreting the results back in the physical world, and (4) evaluating whether the result is adequate or whether the original model needs to be refined.

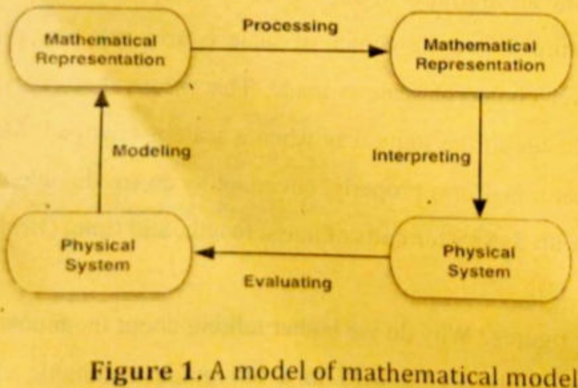

Figure 1. A model of mathematical modeling

Often these all happen at once - are intertwined. (The diagram is not meant to imply a step-by-step algorithmic process). In modern science classes, processing is often stressed and the remaining elements short-changed or ignored. But in modern science, mathematics integrates with our modern science knowledge and does work for us. It lets us carry out chains of reasoning that are longer than we can do in our head, by using formal and logical reasoning represented symbolically

An example of how we use equations to organize and pack our conceptual knowledge is shown in Figure 4: Newton's second law. When we just write "F=ma", our students may see it simply as a way to calculate either F or a and miss the deeper meaning.

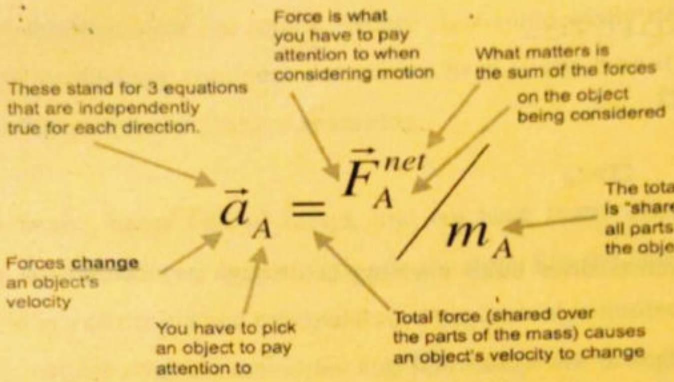

Figure 2. Conceptual knowledge packed in an equation.

3. Application of Non-liner Mathematical Eqations

In modern science, we "make physical meaning" with mathematics. Mathematics is a critical piece of how we decide we know something - our epistemology. What does that mean and how does it work?

To develop an answer, we first ought to consider the question, "How do we make meaning with words?" We'll draw on cognitive semantics – the study of the meaning of words in the intersec-tion of cognitive science and linguistics. Some key ideas developed in these fields are relevant:

- Embodied cognition – Meaning is grounded in physical experience. (Lakoff & Johnson, 1980/2003)
- Encyclopaedic knowledge – Webs of associations build meaning. (Langacker 1987)
- Contextualization – Meaning is constructed dynamically in response to perceived con-text. (Evans & Green 2006)
- Blending – New knowledge can be created by combining and integrating distinct mental spaces. (Fauconnier & Turner 2003)

One way embodiment allows mathematics to feel meaningful in mathematics is with symbolic forms (Sherin 2001, Redish & Kuo 2014): associating symbol structure with relations abstracted from (em-bodied) physical experience

- Parts of a whole: □ = □ + □ + □ ...
- Base + change: □ = □
- ⍰⍰ Balancing: □ = □

A second way mathematics build meaning is through association via multiple representations

- Equations
- Numbers
- Raphs

Physicists tend to make additional meaning of mathematical symbology by associating symbols with physical measurements. This allows connections to physical experience and associations to real world knowledge. And that knowledge may be built up as students learn modern science.

But just as we saw with Introductory students, students at more advahced levels may not apply knowledge they have about the physical world in a math problem. Figure 5 shows an example drawn from an upper division electricity and magnetism class for modern science

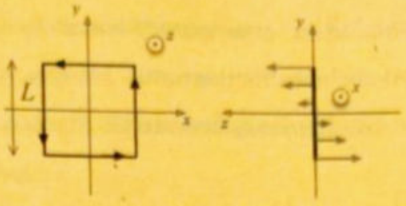

Figure 3. An E&M problem.

majors. Our data is taken from a video of two students working on a problem from their text (Griffiths1999)

The Student A immediately folded his cards in response to student B's more mathematically sophis-ticated reason and agreed she must be right. Both students valued (complex) mathematical rea-soning (where they could easily make a mistake) over a simple and compelling argument (where it's hard to see how it could be wrong) that blends mathematical and physical reasoning.

Our brains know lots of things, and we have many resources for solving our problems, both in life and in a modern science class. But the amount of knowledge that can be held in one's mind and manipulated at any instant is limited (Baddeley 1998). The process by which relevant memories and knowledge are brought to the fore is called framing (Tannen 1994). When that framing is particularly concerned with knowledge building or problem solving, I refer to it as epistemolog-ical framing (Hammer et al. 2005, Bing & Redish 2012).

4. Experimental Case Study

The class is intended to articulate with the rest of these students' curriculum, sr calculus, biology, and chemistry are pre-requisites. This allows us to find places where modern science has authentic val-ue for biology students – places where they have difficulty making sense of important but com-plex issues such as chemical bonding (Dreyfus et al. 2014), diffusion (Moore et al. 2014), or entropy and free energy (Geller et al. 2014). The goals of the course are to (1) create prototype open-source instructional materials that can be shared, (2) focus on interdisciplinary coordina-tion of instruction in biology, chemistry, modern science, and math, and (3) emphasize competency-based instruction, building general scientific skills. Since modern science uses mathematics heavily, it's an appropriate place in the curriculum to emphasize how mathematics are used in science.

The course was built with extensive negotiations among all the relevant disciplines (Redish & Cooke 2013, Redish et al. 2014). One of the important things we learned form these negotia-tions is that the epistemological resources biology students were comfortable using differed from those expected in a modern science class. Some resources common in introductory biology are:

1. Physical intuition: Knowledge built from experience and perception is trustworthy.
2. Life is complex: Living organisms require multiple related processes to maintain life.
3. Categorization and classification: Comparison of related organisms yields insight.
4. By trusted authority: Information from an authoritative source can be trusted.
5. Naming is important: Many distinct components of organisms need to be identified, so learning a large vocabulary is useful.
6. Heuristics: There are broad principles that govern multiple situations.
7. Function implies structure: The historical fact of natural selection leads to strong structure-function relationships.

5. Conclusion

I have presented an analysis of how mathematics is used in modern science, including both an unpacking of what professionals do and an analysis of how students respond. I have shown that this can both can give insight into student difficulties reasoning with mathematics and potentially provide guidance for how to focus on epistemological issues that might create barriers between what a modern science instructor is trying to teach and what the students are learning.

We have developed three ways to talk about how students use knowledge, and mathematical knowledge in particular. (1) Epistemological resources – Generalized categories of "How do we know?" warrants; (2) Epistemological framing - The process of deciding what e-resources are relevant to the current task (NOT necessarily a conscious process); and (3) Epistemological stances - A coherent set of e-resources often activated together.

But be careful! These are NOT intended to describe distinct mental structures. Rather, we use them to emphasize different aspects of what may be a unitary process: activating a subset of the knowledge you have in a particular situation. A warrant focuses on a specific argument, using particular elements of the current context. ("Since the path integral of a conservative force is path independent, these two integrals will have the same value.") A resource focuses on the general class of warrant being used. ("You can trust the results in a reliable source such as a textbook.") Framing focuses attention on the interaction between cue and response.

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